

# Charged and neutral vector meson under magnetic field

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The vector meson  $\rho$  in the presence of external magnetic field has been investigated in the framework of the Nambu–Jona-Lasinio model, where mesons are constructed by infinite sum of quark-loop chains by using random phase approximation. The  $\rho$  meson polarization function is calculated to the leading order of  $1/N_c$  expansion. It is found that the constituent quark mass increases with magnetic field, the masses of the neutral vector meson  $\rho^0$  with spin component  $s_z = 0, \pm 1$  and the charged vector meson  $\rho^\pm$  with  $s_z = 0$  also increases with magnetic field. However, the mass square of the charged vector meson  $\rho^\pm$  ( $\rho^\pm$ ) with  $s_z = +1$  ( $s_z = -1$ ) decreases linearly with magnetic field and drops to zero at the critical magnetic field  $eB_c \simeq 0.2\text{GeV}^2$ , which indicates the possible condensation of charged vector meson in the vacuum. This critical magnetic field is much lower than the value  $eB_c = 0.6\text{GeV}^2$  predicted by a point-like vector meson. We also show that if we use lowest Landau level approximation, the mass of the charged vector meson  $\rho^\pm$  for  $s_z = \pm 1$  cannot drop to zero at high magnetic fields.

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## I. INTRODUCTION

Strong magnetic fields with strength of  $10^{18} \sim 10^{20}\text{G}$  (corresponding to  $eB \sim (0.1 - 1.0\text{ GeV})^2$ ), can be generated in the laboratory through non-central heavy ion collisions [1, 2] at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC). In the surface of magnetars, magnetic fields can reach  $10^{14-15}\text{G}$ , which is thousand of times larger than that of an average pulsar, and in the inner core of magnetars the magnetic fields could reach as high as  $10^{18} \sim 10^{20}\text{G}$ . Therefore, it is important to understand the properties of Quantum Chromodynamics (QCD) vacuum and hot/dense quark matter under strong magnetic fields. Many progresses have been made in this field, for example, the Chiral Magnetic Effect (CME) [3–5], Chiral Vortical Effect (CVE) [6], the Magnetic Catalysis and Inverse Magnetic Catalysis [7], and the Vacuum Superconductor [8, 9]. For reviews please refer to Refs. [10, 11].

The heavy ion collision experiment provides a unique environment to search for the possibility of local parity violation and anomalous transport effects. The QCD vacuum has a non-trivial topological structure, at high temperatures, because sphaleron transitions between distinct classical vacua cause an imbalance between the number of quarks with different chirality, and results in a violation of the  $\mathcal{P}$ - and  $\mathcal{CP}$ -symmetry. In the presence of a strong magnetic field, an electromagnetic current can be generated along the magnetic field, which is called the anomalous Chiral Magnetic Effect (CME) [3–5], and will induce the Charge Separation Effect. Recently, the ob-

servation of charge azimuthal correlations [12, 13] from RHIC and LHC possibly resulting from the anomalous Chiral Magnetic Effect (CME).

Chiral symmetry breaking and restoration under strong magnetic field has been discussed for many years. It has been recognized since 1990s' that the chiral condensate increases thus the critical temperature of chiral phase transition should also increase with magnetic field, which is the well-known magnetic catalysis [14–16]. However, the lattice group [7] observed the inverse magnetic catalysis around  $T_c$ , which turns out to be a surprise and puzzle. There have been several proposals [17–21] trying to understand the underlying mechanism of inverse magnetic catalysis. It is proposed that the chirality imbalance induced by sphaleron transition [20] or instanton–anti-instanton molecule pairing [21] can naturally explain the inverse magnetic catalysis near  $T_c$ .

The vacuum superconductor was firstly proposed in hadronic models [8] based on the energy of a free particle under magnetic fields by neglecting the internal structure of vector mesons. More calculations have been done by using different models in Refs.[22–25] and Refs.[26–28], however, whether there exists vacuum superconductor is still under debate.

Considering a free charged relativistic particle with mass  $m$ , electric charge  $q$  and spin  $s$ , moving in a homogeneous background of an external magnetic field  $B$  directed along the  $z$  axis, the relativistic energy levels  $\varepsilon$  of this particle are given by the following formula:

$$\varepsilon_{n,s_z}^2(p_z) = p_z^2 + (2n - 2\text{sgn}(q)s_z + 1)|qB| + m^2, \quad (1)$$

where  $n \geq 0$  is the Landau level,  $s_z = -s, \dots, s$  is the projection of the spin  $s$ , and  $p_z$  is the particle's momentum along the magnetic field. For vector meson  $\rho$  with spin  $s = 1$ , its minimal effective mass square corresponding to the lowest energy state of (1) with  $p_z = 0$ , thus

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has the form of

$$M_{\rho^\pm}^2(B) = m_{\rho^\pm}^2 - eB. \quad (2)$$

The vacuum masses of the  $\rho^\pm$  mesons  $m_\rho = 775.5 \text{ MeV}$  implies that the lowest energy of the charged  $\rho$ -meson in the external magnetic field may become purely imaginary if the magnetic field exceeds the following critical value

$$eB_c = m_\rho^2 \simeq 0.6 \text{ GeV}^2. \quad (3)$$

This indicates that around this critical magnetic field, the vector mode will become unstable, therefore there should appear electric charged vector meson condensation in the vacuum, which is called the vacuum superconductor [8].

However, it is still not clear whether charged vector meson condensation can happen, because different methods give different answers.

It was argued in [22] that due to Vafa-Witten theorem, charged vector meson condensation cannot occur in QCD in a strong magnetic field because any global-internal symmetry is not spontaneously broken by a magnetic field. Then it was pointed out in [25], that the Vafa-Witten theorem would not forbid the charged  $\rho$  condensation under external magnetic fields.

However, the authors in [22] performed lattice calculation and their results showed that the charged  $\rho$  meson mass firstly decreases with magnetic field and has a minimum around  $eB \simeq 1 \text{ GeV}^2$ , then again increases with magnetic field, but the mass will not decrease to zero. This result is confirmed in Ref.[27] by solving the meson spectra in a relativistic quark-antiquark system using the relativistic Hamiltonian technique. Also, in [28], the author obtained similar results in the framework of Dyson-Schwinger equations.

On the other hand, the charged  $\rho$  condensation was confirmed in SU(2) lattice calculation in [29], and from calculations in the Nambu-Jona-Lasinio (NJL) model [9, 26] as well as from gauge/gravity correspondence [30, 31]. However, the critical magnetic field for charged  $\rho$  mass becoming zero are different for different calculations. The hadronic model gives  $eB_c = 0.6 \text{ GeV}^2$ , the estimation by using NJL model in [9] gives  $eB_c > 1.0 \text{ GeV}^2$ , while the NJL model calculation in [26] gives  $eB_c \simeq 0.98 M_q^2$ .

The motivation of our work is to investigate the vector meson carefully in the NJL model and try to understand different results in different models. The paper is organized as follows. In Sec. II, we give a general description of the NJL model under magnetic field including the effective four-quark interaction in the vector channel. In Sec. III, we introduce how to calculate the vector meson mass under magnetic field. We give our numerical results and analysis in Sec. IV and then in Sec. V we give the conclusion and discussion.

## II. THE SU(2) NJL MODEL UNDER MAGNETIC FIELD

We investigate the properties of vector meson under magnetic fields in the framework of the SU(2) Nambu-Jona-Lasinio (NJL) model [32–37]. The Lagrangian density of our model is given by

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i \not{D} - \hat{m})\psi + G_S [(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\vec{\tau}\psi)^2] \\ & - G_V [(\bar{\psi}\gamma^\mu\tau^a\psi)^2 + (\bar{\psi}\gamma^\mu\gamma^5\tau^a\psi)^2] \\ & - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \end{aligned} \quad (4)$$

Where  $\psi$  corresponds to the quark field of two light flavors u and d,  $\hat{m} = \text{diag}(m_u, m_d)$  is the current quark mass matrix of u and d quarks,  $\tau^a = (I, \vec{\tau})$  with  $\vec{\tau} = (\tau^1, \tau^2, \tau^3)$  representing the isospin Pauli matrices, and  $G_S$  and  $G_V$  are the coupling constants with respect to the scalar (pseudoscalar) and the vector (axial-vector) channels, respectively. The covariant derivative,  $D_\mu = \partial_\mu - iq_f A_\mu^{ext}$ , couples quarks to an external magnetic field  $\mathbf{B} = (0, 0, B)$  along the positive  $z$  direction via a background field, for example,  $A_\mu^{ext} = (0, 0, Bx, 0)$ .  $q_f = (-1/3, 2/3)$  is defined as the electric charge of the quark field. The field strength tensor  $F_{\mu\nu}$  is defined as usual by  $F_{\mu\nu} = \partial_\mu A_\nu^{ext} - \partial_\nu A_\mu^{ext}$ , with  $A_\mu^{ext}$  fixed as above.

The above Lagrangian is equivalent to the semi-bosonized Lagrangian

$$\begin{aligned} \mathcal{L}_{sb} = & \bar{\psi}(x)(i\gamma^\mu D_\mu - m_0)\psi(x) - \bar{\psi}(\sigma + i\gamma_5\vec{\tau} \cdot \vec{\pi})\psi \\ & - \frac{(\sigma^2 + \vec{\pi}^2)}{4G_S} + \frac{(V_\mu^a V^{a\mu} + A_\mu^a A^{a\mu})}{4G_V} - \frac{B^2}{2}, \end{aligned} \quad (5)$$

where the Euler-Lagrange equations of motion for the auxiliary fields lead to the constraints

$$\sigma(x) = -2G_S \bar{\psi}(x)\psi(x), \quad (6)$$

$$\vec{\pi}(x) = -2G_S \bar{\psi}(x)i\gamma_5\vec{\tau}\psi(x), \quad (7)$$

$$V_\mu^a(x) = -2G_V \bar{\psi}(x)\gamma_\mu\tau^a\psi(x), \quad (8)$$

$$A_\mu^a(x) = -2G_V \bar{\psi}(x)\gamma_\mu\gamma^5\tau^a\psi(x). \quad (9)$$

In the vacuum, the quark-antiquark condenses and quarks obtain dynamical masses, so the constituent quark mass  $M$  of u,d quarks is defined by

$$M = m_0 - 2G_S <\bar{\psi}\psi>, \quad (10)$$

where we assume  $m_u = m_d = m_0$ . The inverse quark propagator takes the form of

$$iS_Q^{-1}(\sigma, \vec{\pi}) \equiv i\gamma^\mu D_\mu - (\hat{m} + \sigma + i\gamma^5\vec{\tau} \cdot \vec{\pi}). \quad (11)$$

Under the external magnetic field  $\mathbf{B} = (0, 0, eB)$ , the solution of the Dirac equation with a constant magnetic field is known, which forms the complete set of orthogonal wave-functions, and the Ritus fermion propagator takes the form of [38–40]

$$S_Q(x, y) = i \sum_{\vec{p}=0}^{\infty} \mathcal{D}_{\vec{p}} e^{-i\vec{p} \cdot (x-y)} P_p(x_1) D_Q^{-1}(\vec{p}) P_p(y_1). \quad (12)$$

Where

$$D_Q(\bar{p}) = \gamma \cdot \bar{p}_Q - M, \quad (13)$$

with  $M$  the constituent quark mass given in Eq.(10), and  $\mathcal{D}\tilde{p} \equiv \frac{dp_0 dp_2 dp_3}{(2\pi)^3}$ ,  $\tilde{p} = (p_0, 0, p_2, p_3)$  and  $\bar{p}$  is the Ritus momentum

$$\bar{p} = (p_0, 0, -s_Q \sqrt{2|QeB|}p, p_3),$$

with  $s_Q \equiv \text{sgn}(QeB)$ , here  $Q$  is the diagonal matrix, and we will use  $s \equiv \text{sgn}(qfeB)$  for the elements of this  $2 \times 2$  matrix  $s_Q$ . The projection matrix with respect to the Dirac index according to Ritus method is given below:

$$P_p(x_1) \equiv \frac{1}{2}[f_p^{+s}(x_1) + \Pi_p f_p^{-s}(x_1)] + \frac{is_Q}{2}[f_p^{+s}(x_1) - \Pi_p f_p^{-s}(x_1)]\gamma^1\gamma^2. \quad (14)$$

Here,  $\Pi_p \equiv 1 - \delta_{p,0}$  considers the spin degeneracy in the lowest Landau level with  $p = 0$ . Moreover,  $f_p^{\pm s}(x_1)$  are defined by

$$\begin{aligned} f_p^{+s}(x_1) &\equiv \phi_p(x_1 - s_Q p_2 \ell_B^2), & p &= 0, 1, 2, \dots, \\ f_p^{-s}(x_1) &\equiv \phi_{p-1}(x_1 - s_Q p_2 \ell_B^2), & p &= 1, 2, 3, \dots, \end{aligned} \quad (15)$$

where  $\phi_p(x)$  is a function of Hermite polynomials  $H_p(x)$  in the form

$$\phi_p(x) \equiv a_p \exp\left(-\frac{x^2}{2\ell_B^2}\right) H_p\left(\frac{x}{\ell_B}\right). \quad (16)$$

Here,  $a_p \equiv (2^p p! \sqrt{\pi} \ell_B)^{-1/2}$  is the normalization factor and  $\ell_B \equiv |QeB|^{-1/2}$  is the magnetic length.

### III. VECTOR MESON MASS UNDER MAGNETIC FIELD

In the framework of the NJL model, mesons are  $q\bar{q}$  bound states or resonances and can be obtained from the quark-antiquark scattering amplitude [41, 42]. In the random phase approximation, the full propagator of  $\rho$  meson shown in Fig. 1 can be expressed to leading order in  $1/N_c$  as an infinite sum of quark-loop chains, and can also be recast into the form of a Schwinger-Dyson equation. The  $\rho$ -meson propagator  $D_{ab}^{\mu\nu}(q^2)$  can be obtained from one-loop polarization function  $\Pi_{\mu\nu,ab}(q^2)$  shown in Fig. 2 via the Schwinger-Dyson equation and takes the form of

$$[-iD_{ab}^{\mu\nu}] = [-2iG_V \delta_{ab} g^{\mu\nu}] + [-2iG_V \delta_{ac} g^{\mu\lambda}] [-i\Pi_{\lambda\sigma,cd}] [-iD_{db}^{\sigma\nu}], \quad (17)$$

where  $a, b, c, d$  are isospin indices, and  $\mu, \nu$  Lorentz indices.

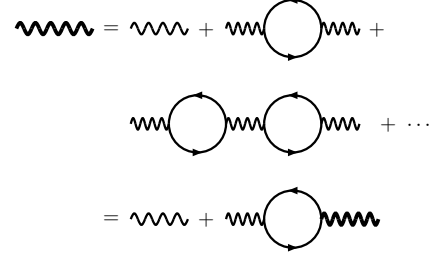


FIG. 1: The full propagator of  $\rho$  meson in the random phase approximation (RPA). Thick wavy lines indicate the full propagator  $D_{ab}^{\mu\nu}$  of  $\rho$ -meson, and thin wavy lines the bare propagator  $-2G_V \delta_{ab}$ .

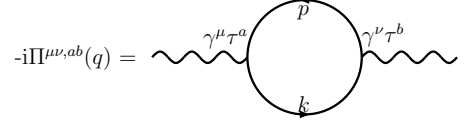


FIG. 2: The  $\rho$  meson polarization function  $\Pi^{\mu\nu,ab}$  with one quark loop contribution, i.e., the leading order contribution in  $1/N_c$  expansion.

The one quark loop polarization function  $\Pi_{\mu\nu,ab}(q^2)$  takes the form of

$$\Pi^{\mu\nu,ab}(q^2) = i \sum_{p,k=0}^{\infty} \int \mathcal{D}\tilde{p} \mathcal{D}\tilde{k} \int d^4x e^{-i(\tilde{p}-\tilde{k}-q)\cdot x} \Lambda_{pk}^{\mu\nu,ab}(\tilde{p}, \tilde{k}, x_1), \quad (18)$$

where

$$\begin{aligned} \Lambda_{pk}^{\mu\nu,ab}(\tilde{p}, \tilde{k}, x_1) &= \text{tr}_{sf c} [\gamma^\mu \tau^a P_p(x_1) D_Q^{-1}(\tilde{p}) P_p(0) \gamma^\nu \tau^b \\ &\quad \times K_k(0) D_Q^{-1}(\tilde{k}) K_k(x_1)] \end{aligned} \quad (19)$$

with external momentum  $q = p - k$ .

#### A. Charged $\rho^\pm$ meson

For charged  $\rho^\pm$  meson, the isospin Pauli matrices take  $\tau^a = \tau^\pm$  and  $\tau^b = \tau^\mp$ , respectively, with

$$\tau^\pm = \frac{1}{\sqrt{2}}(\tau^1 \pm \tau^2). \quad (20)$$

In the rest frame,  $q_\mu = (M_{\rho^\pm}, \mathbf{0})$ , the polarization function has the form of

$$\Pi_{\rho^\pm}^{\mu\nu}(q^2) = i \sum_{p,k=0}^{\infty} \int \frac{dk_0 dk_3}{(2\pi)^3} \int dk_2 dx_1 \Lambda_{\rho^\pm, pk}^{\mu\nu}(\tilde{p}, \tilde{k}, x_1), \quad (21)$$

where

$$\Lambda_{\rho^\pm, pk}^{\mu\nu}(\bar{p}, \bar{k}, x_1) = Tr_{scf}[\gamma^\mu \tau^\pm P_p(x_1) D_q^{-1}(\bar{p}) P_p(0) \gamma^\nu \tau^\mp K_k(0) D_Q^{-1}(\bar{k}) K_k(x_1)]. \quad (22)$$

In the following, we calculate the components of polarization tensor for  $\rho^\pm$ .  $\mu, \nu = 1, 2$  correspond to the transverse components of the polarization tensor and we introduce the definition  $\mu_\perp \nu_\perp$ . Thus,

$$\Lambda_{\rho^\pm, pk}^{\mu_\perp \nu_\perp} = Tr_{scf}[(A^+ - i s_p \gamma^1 \gamma^2 A^-) \gamma^\mu D_Q^{-1}(\bar{p}) \gamma^\nu (\alpha^+ - i s_p \gamma^1 \gamma^2 \alpha^-) D_Q^{-1}(\bar{k})], \quad (23)$$

where

$$A^\pm = \frac{1}{2}(f_p^{+s_p}(x_1) f_k^{+s_k}(x_1) \pm \Pi_p \Pi_k f_p^{-s_p}(x_1) f_k^{-s_k}(x_1))$$

$$\alpha^\pm = \frac{1}{2}(f_p^{+s_p}(0) f_k^{+s_k}(0) \pm \Pi_p \Pi_k f_p^{-s_p}(0) f_k^{-s_k}(0))$$

and  $s_p = \text{sgn}(q_{fp} e B)$  ( $q_{fp}$  is the electric charge of u quark for  $\rho^+$ , and  $q_{fp}$  is the electric charge of d quark for  $\rho^-$ ).

The components can be derived and have the following forms:

$$\Lambda_{\rho^\pm, pk}^{11} = 4N_c N_f \frac{1}{(p_0^2 - \omega_p^2)(k_0^2 - \omega_k^2)} [(\bar{p}\bar{k} - M^2) A^+ \alpha^+ + (2\bar{k}_2 \bar{p}_2 + \bar{p}\bar{k} - M^2) A^- \alpha^-], \quad (24)$$

$$\Lambda_{\rho^\pm, pk}^{12} = 4N_c N_f \frac{1}{(p_0^2 - \omega_p^2)(k_0^2 - \omega_k^2)} [(\bar{p}\bar{k} + 2\bar{p}_2 \bar{k}_2 - M^2)(-i s_p A^- \alpha^+) + (\bar{p}\bar{k} - M^2)(-i s_p A^+ \alpha^-)], \quad (25)$$

$$\Lambda_{\rho^\pm, pk}^{21} = 4N_c N_f \frac{1}{(p_0^2 - \omega_p^2)(k_0^2 - \omega_k^2)} [(-\bar{p}\bar{k} + M^2)(-i s_p A^- \alpha^+) + (-\bar{p}\bar{k} - 2\bar{p}_2 \bar{k}_2 + M^2)(-i s_p A^+ \alpha^-)], \quad (26)$$

$$\Lambda_{\rho^\pm, pk}^{22} = 4N_c N_f \frac{1}{(p_0^2 - \omega_p^2)(k_0^2 - \omega_k^2)} [(\bar{p}\bar{k} - M^2) A^- \alpha^- + (2\bar{p}_2 \bar{k}_2 + \bar{p}\bar{k} - M^2) A^+ \alpha^+], \quad (27)$$

where the  $\omega_p^2 = 2|q_{fp} e B|p + k_3^2 + M^2$ .

For  $\mu, \nu = 0, 3$ , they correspond to the longitudinal components of the polarization tensor and are defined by  $\mu_\parallel \nu_\parallel$ .

$$\Lambda_{\rho^\pm, pk}^{\mu_\parallel \nu_\parallel} = Tr_{scf}[(B^+ - i s_p \gamma^1 \gamma^2 B^-) \gamma^\mu D_Q^{-1}(\bar{p}) \gamma^\nu (\beta^+ - i s_p \gamma^1 \gamma^2 \beta^-) D_Q^{-1}(\bar{k})], \quad (28)$$

where

$$B^\pm = \frac{1}{2}[\Pi_p f_k^{s_k}(x_1) f_p^{-s_p}(x_1) \pm \Pi_k f_p^{s_p}(x_1) f_k^{-s_k}(x_1)]$$

$$\beta^\pm = \frac{1}{2}[\Pi_p f_k^{s_k}(0) f_p^{-s_p}(0) \pm \Pi_k f_p^{s_p}(0) f_k^{-s_k}(0)].$$

The component

$$\Lambda_{\rho^\pm, pk}^{33} = 4N_c N_f \frac{1}{(p_0^2 - \omega_p^2)(k_0^2 - \omega_k^2)} [(p_0 k_0 - \bar{p}_2 \bar{k}_2 + k_3^2 - M^2) B^+ \beta^+ + (p_0 k_0 + \bar{p}_2 \bar{k}_2 + k_3^2 - M^2) B^- \beta^-]. \quad (29)$$

The other matrix elements of  $\Pi_{\rho^\pm}^{\mu\nu}$  are zero.

Finally, we can get the matrix

$$\Pi_{\rho^\pm}^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \Pi^{11} & \Pi^{12} & 0 \\ 0 & \Pi^{21} & \Pi^{22} & 0 \\ 0 & 0 & 0 & \Pi^{33} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & -ib & 0 \\ 0 & ib & a & 0 \\ 0 & 0 & 0 & c \end{pmatrix}, \quad (30)$$

where we have used relations  $\Pi^{11} = \Pi^{22} = a$  and  $\Pi^{12} = -\Pi^{21} = ib$ .

The polarization tensor can be decomposed as following

$$\Pi_{\rho^\pm}^{\mu\nu} = [A_1^2 P_1^{\mu\nu} + A_2^2 P_2^{\mu\nu} + A_3^2 L^{\mu\nu} + A_4^2 u^\mu u^\nu], \quad (31)$$

where  $u^\mu = (1, 0, 0, 0)$  is the four momentum in the rest frame, and we have introduced the spin projection operator

$$P_1^{\mu\nu} = -\epsilon_1^\mu \epsilon_1^\nu, (s_z = -1 \text{ for } \rho^\pm), \quad (32)$$

$$P_2^{\mu\nu} = -\epsilon_2^\mu \epsilon_2^\nu, (s_z = 1 \text{ for } \rho^\pm), \quad (33)$$

$$L^{\mu\nu} = -b^\mu b^\nu, (s_z = 0 \text{ for } \rho^\pm). \quad (34)$$

Here the right and left-handed polarization vectors

$$\epsilon_1^\mu = \frac{1}{\sqrt{2}}(0, 1, i, 0), \quad (35)$$

$$\epsilon_2^\mu = \frac{1}{\sqrt{2}}(0, 1, -i, 0), \quad (36)$$

are parallel or anti-parallel to the the external magnetic field direction  $b^\mu = (0, 0, 0, 1)$ .

Consequently, the charged  $\rho^\pm$  meson propagator can be written as:

$$D_{\rho^\pm}^{\mu\nu}(q^2) = [D_1(q^2) P_1^{\mu\nu} + D_2(q^2) P_2^{\mu\nu} + D_3(q^2) L^{\mu\nu} + D_4(q^2) u^\mu u^\nu]. \quad (37)$$

Each component  $D_i$  can be written in the form of

$$D_i(q^2) = \frac{2G_V}{1 + 2G_V A_i^2} \quad (38)$$

by using RPA, and the mass of charged  $\rho^\pm$  can be determined by the gap equation:

$$1 + 2G_V A_i^2 = 0. \quad (39)$$

For charged  $\rho^\pm$  meson with spin components  $s_z = 1, 0, -1$ , we have gap equations:

$$1 + 2G_V A_1^2 = 0, (s_z = -1), \quad (40)$$

$$1 + 2G_V A_2^2 = 0, (s_z = 1), \quad (41)$$

$$1 + 2G_V A_3^2 = 0, (s_z = 0). \quad (42)$$

From Eq.(30), it is easy to find that

$$A_1^2 = -(a + b), \quad (43)$$

$$\begin{aligned} A_2^2 &= b - a, \\ A_3^2 &= c. \end{aligned} \quad (44)$$

In the rest frame of  $\rho$  with  $q^\mu = (M_{\rho^\pm}, \mathbf{0})$ , it is easy to find  $A_4^2 = 0$ , which is required by the Ward identity.

### B. Neutral $\rho^0$ meson

For charge neutral  $\rho^0$  meson, the isospin Pauli matrices take  $\tau^a = \tau^3$  and  $\tau^b = \tau^3$ . In the rest frame,  $q_\mu = (M_{\rho^0}, \mathbf{0})$ , the polarization function has the form of

$$\begin{aligned} \Pi_{\rho^0}^{\mu\nu}(q^2) &= i \sum_{p,k=0}^{\infty} \int \frac{dk_0 dk_3}{(2\pi)^3} T_{\rho^0}^{\mu\nu}(\bar{p}, \bar{k}), \\ T_{\rho^0}^{\mu\nu}(\bar{p}, \bar{k}) &= \int dk_2 dx_1 \Lambda_{\rho^0, pk}^{\mu\nu}, \end{aligned} \quad (45)$$

with

$$\begin{aligned} \Lambda_{\rho^0, pk}^{\mu\nu}(\bar{p}, \bar{k}, x_1) &= Tr_{sf}[\gamma^\mu \tau^3 P_p(x_1) D_q^{-1}(\bar{p}) P_p(0) \\ &\quad \gamma^\nu \tau^3 K_k(0) D_q^{-1}(\bar{k}) K_k(x_1)]. \end{aligned} \quad (46)$$

By using the orthonormality relations of  $f_p^{\pm s}$ :

$$\int dx_1 f_p^{+s}(x) f_k^{+s}(x) \Big|_{p_2=k_2} = \delta_{pk}, \quad (47)$$

and

$$\int dk_2 f_p^{+s}(0) f_k^{+s}(0) \Big|_{p_2=k_2} = \frac{\delta_{pk}}{\ell_B^2}, \quad (48)$$

$T_{\rho^0}^{\mu\nu}$  can be simplified as

$$\begin{aligned} T_{\rho^0}^{11}(\bar{p}, \bar{k}) &= T_{\rho^0}^{22}(\bar{p}, \bar{k}) = 4N_c \sum_{q_f \in \{\frac{-1}{3}, \frac{2}{3}\}} |q_f eB| \\ &\times \frac{p_0 k_0 - k_3^2 - M^2}{(p_0^2 - \omega_p^2)(k_0^2 - \omega_k^2)} \left( \frac{1}{2} \delta_{k,p-1} + \frac{1}{2} \delta_{p,k-1} \right), \end{aligned} \quad (49)$$

$$\begin{aligned} T_{\rho^0}^{33}(\bar{p}, \bar{k}) &= 2N_c \sum_{q_f \in \{\frac{-1}{3}, \frac{2}{3}\}} |q_f eB| \\ &\times \frac{p_0 k_0 - \bar{p}_2 \bar{k}_2 + k_3^2 - M^2}{(p_0^2 - \omega_p^2)(k_0^2 - \omega_k^2)} \alpha_k \delta_{p,k}, \end{aligned} \quad (50)$$

where  $\alpha_k = 2 - \delta_{k,0}$ .

We get the matrix

$$\Pi_{\rho^0}^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \Pi_{\rho^0}^{11} & 0 & 0 \\ 0 & 0 & \Pi_{\rho^0}^{22} & 0 \\ 0 & 0 & 0 & \Pi_{\rho^0}^{33} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & d & 0 \\ 0 & 0 & 0 & e \end{pmatrix}, \quad (51)$$

here we have used the relations  $\Pi_{\rho^0}^{11} = \Pi_{\rho^0}^{22} = d$  and  $\Pi_{\rho^0}^{33} = e$ .

Similar to the case of charged  $\rho^\pm$  mesons, charge neutral  $\rho^0$  meson with spin components  $s_z = 1, 0, -1$  take the following gap equations:

$$1 + 2G_V A_1^2 = 0, \quad (s_z = -1), \quad (52)$$

$$1 + 2G_V A_2^2 = 0, \quad (s_z = 1), \quad (53)$$

$$1 + 2G_V A_3^2 = 0, \quad (s_z = 0), \quad (54)$$

From Eq.(51), it is easy to find that

$$A_1^2 = -d, \quad (55)$$

$$\begin{aligned} A_2^2 &= -d, \\ A_3^2 &= e. \end{aligned} \quad (56)$$

## IV. NUMERICAL RESULTS

Following Ref.[41], the model parameters are fitted by reproducing the pion decay constant, the vacuum quark mass, the mass of  $\pi$  and the mass of  $\rho$  in the vacuum. They are given by  $\Lambda = 582\text{MeV}$ ,  $G_S \Lambda^2 = 2.388$ ,  $G_V \Lambda^2 = 1.73$ . These parameters correspond to  $f_\pi = 95\text{MeV}$ ,  $m_\pi = 140\text{MeV}$ ,  $M_\rho = 768\text{MeV}$ , the vacuum condensation  $\langle \bar{u}u \rangle = -(252)^3\text{MeV}^3$ , the vacuum quark mass  $M = 458\text{MeV}$  and the current quark mass  $m_0 = 5\text{MeV}$ . We use the soft cut-off function as in [43]

$$f_\Lambda = \frac{\Lambda^{10}}{\Lambda^{10} + \mathbf{k}^{2*5}}, \quad (57)$$

$$f_{\Lambda, eB} = \frac{\Lambda^{10}}{\Lambda^{10} + (k_z^2 + 2|Q_f eB|k)^5}, \quad (58)$$

for zero and nonzero magnetic fields.

### A. Masses of charged and neutral $\rho$ mesons

We calculate the vector meson mass numerically, and the quark mass  $M$  is also solved self-consistently from the gap equation Eq. (10). It can be read from Fig. 3, the constituent quark mass increases with magnetic field, which is the well-known magnetic catalysis effect [16].

The numerical results for the mass square of charged  $\rho^\pm$  with spin component  $s_z = \pm 1$  is shown in Fig. 4. For numerical calculations, we have summed 20 Landau levels, the results for summation above 10 Landau levels are saturated. It is found that even though the constituent quark mass increases with magnetic field, the mass square of charged  $\rho^\pm$  with  $s_z = \pm 1$  decreases with  $eB$  linearly  $M_{\rho^\pm}^2(B) = m_{\rho^\pm}^2 - \kappa eB$  with the slope  $\kappa \approx 3$ , and the mass square goes to zero at  $eB_c \simeq 0.2\text{GeV}^2$ . In Appendix B, by performing the weak magnetic expansion we obtain  $M_{\rho^\pm}^2(B) = m_{\rho^\pm}^2 - \kappa eB$  with  $\kappa_w \approx 1.92$ . For both cases, the mass square of charged  $\rho^\pm$  with spin component  $s_z = \pm 1$  decreases faster with  $eB$  than the

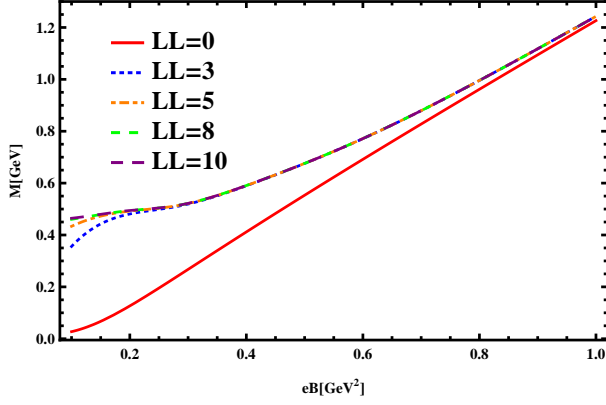


FIG. 3: Quark constituent mass  $M$  as a function of  $eB$  with different Landau levels included in the numerical calculations.

case of free charged relativistic particle, where  $M_{\rho^\pm}^2(B) = m_{\rho^\pm}^2 - \kappa eB$  with  $\kappa = 1$  which gives the critical magnetic field  $eB_c \approx 0.6\text{GeV}^2$ .

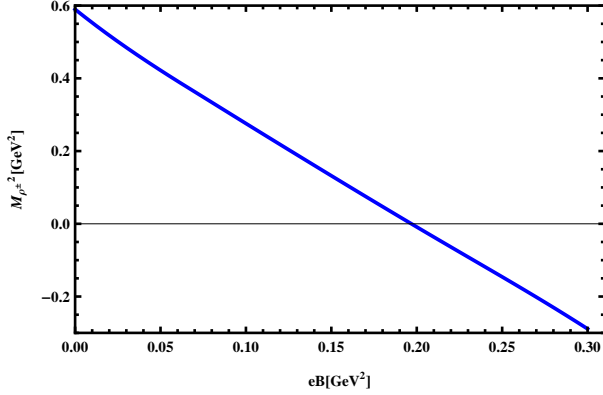


FIG. 4: The mass square of charged  $\rho^\pm$  with spin component  $s_z = \pm 1$  as a function of  $eB$ .

Fig. 5 shows the numerical results for masses of charged vector meson  $\rho^\pm$  with  $s_z = 0$  and neutral vector meson  $\rho^0$  with  $s_z = 0, \pm 1$  as functions of magnetic field  $eB$ . It is found that for all these modes, the masses of vector mesons increase with magnetic field. The neutral vector meson  $\rho^0$  with  $s_z = \pm 1$  increase fast with the magnetic field. The neutral vector meson  $\rho^0$  with  $s_z = 0$  almost remains as a constant at low magnetic field, but then slowly increases with the magnetic field when  $eB > 0.3\text{GeV}^2$ . The lattice group [44] found that the neutral vector meson  $\rho^0$  with  $s_z = \pm 1$  also increase with the magnetic field, however, their result on neutral vector meson  $\rho^0$  with  $s_z = 0$  decreases with the magnetic field.

### B. Mass of charged $\rho^\pm$ with $s_z = \pm 1$ at LLL

As mentioned in the Introduction that there some other results showed that the mass of the charged vector

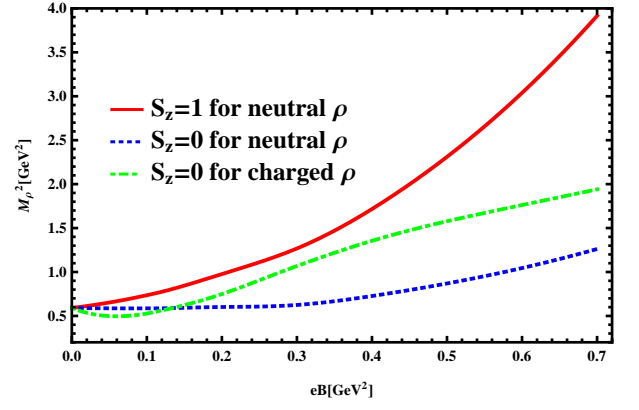


FIG. 5: Masses of charged vector meson  $\rho^\pm$  with  $s_z = 0$  and neutral vector meson  $\rho^0$  with  $s_z = 0, \pm 1$  as functions of magnetic field  $eB$ .

meson  $\rho^\pm$  can never drop to zero at high magnetic field. For example, [22] performed lattice calculation and their results showed that the charged  $\rho$  meson mass firstly decreases with magnetic field and has a minimum around  $eB \simeq 1\text{GeV}^2$ , then again increases with magnetic field, and this result was confirmed in Ref.[27] by solving the meson spectra in a relativistic quark-antiquark system using the relativistic Hamiltonian technique. Also, in [28], the author obtained similar result in the framework of Dyson-Schwinger equations.

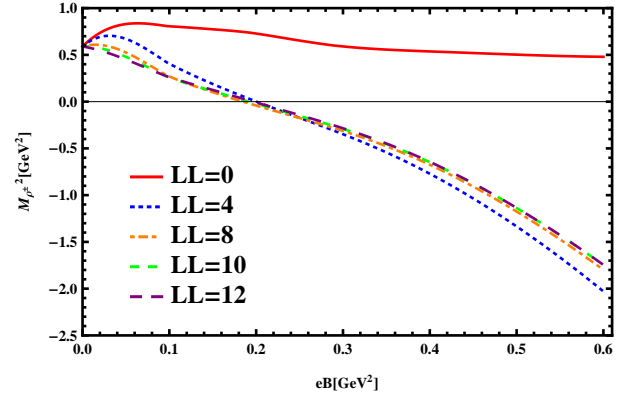


FIG. 6:  $M_{\rho^\pm}^2$  for  $s_z = 1$  as a function of  $eB$  with different Landau levels, with LL the Landau Levels.

In this section, we analyze possible reasons. We don't know the detailed calculations from lattice in Ref.[22] and DSE Ref.[28]. However, the authors used lowest Landau level approximation in Ref.[27]. As we all know, when the magnetic field becomes strong, there exists dimensional reduction  $D \rightarrow D - 2$ , and in strong magnetic field limit, lowest Landau level (LLL) approximation is widely used. From our numerical results in Fig.4, the charged  $\rho^\pm$  for  $s_z = 1$  drops to zero at  $eB_c = 0.2\text{GeV}^2$ , which is not in the range of strong magnetic field. Therefore, we investigate how LLL approximation will affect the mass of  $\rho$  meson.

Fig. 3 shows the constituent quark mass with different Landau levels, where  $LL = 0$  indicates the LLL approximation result. It is observed that at weak magnetic field, the LLL approximation gives a rather small constituent quark mass and cannot describe the spontaneous chiral symmetry breaking. At high magnetic fields, the LLL approximation gives almost the same constituent quark mass as summing over higher Landau levels.

The mass square of charged  $\rho^\pm$  for  $s_z = \pm 1$  with different Landau levels is given in Fig. 6. Contrary to the case of constituent quark mass, at weak magnetic field, LLL approximation gives similar results of  $M_{\rho^\pm}^2(s_z = \pm 1)$  comparing with higher Landau level summation results. However, when the magnetic field increases, the difference between  $M_{\rho^\pm}^2(s_z = \pm 1)$  at LLL approximation and higher LL summation becomes larger and larger. It is noticed that  $M_{\rho^\pm}^2(s_z = \pm 1)$  at LLL approximation changes flatly with the increase of magnetic field and does not drop to zero! This result qualitatively agrees with the results in Ref.[22, 27, 28].

Let's further analyze why  $M_{\rho^\pm}^2(s_z = \pm 1)$  does not drop to zero by using LLL approximation in our framework. For  $\rho^+(s_z = 1)$ , the mass is solved from the gap equation  $1 - 2G_V(a + b) = 0$ . We plot  $a$  and  $b$  for  $M_{\rho^+} = 768\text{MeV}$  in Fig. 7 as functions of  $eB$  with Landau levels  $LL=0$  and 20, respectively. It is found that at weak magnetic field, the LLL approximation only gives 1/5 contribution to  $a$  and 1/2 contribution to  $b$ . At high magnetic field, the LLL approximation gives almost 1/2 contribution to both  $a$  and  $b$ .

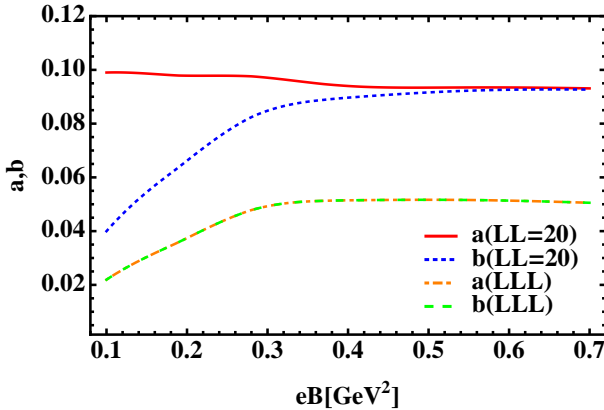


FIG. 7:  $a, b$  as functions of  $eB$  with Landau levels  $LL=0$  and 20, respectively. We have taken  $M_{\rho^+} = 768\text{MeV}$ .

We set  $M_{\rho^+} = 0$ , and in Fig. 8 we plot the gap function  $1 - 2G_V(a + b)$  as a function of  $eB$  with Landau levels  $LL=0$  and 20, respectively. We can see that the gap function at LLL approximation does not cross zero axis, however, the gap function at  $LL=20$  crosses zero axis at  $eB_c \simeq 0.2\text{GeV}^2$ , which is exactly the results given by solving the pole mass from the gap equation.

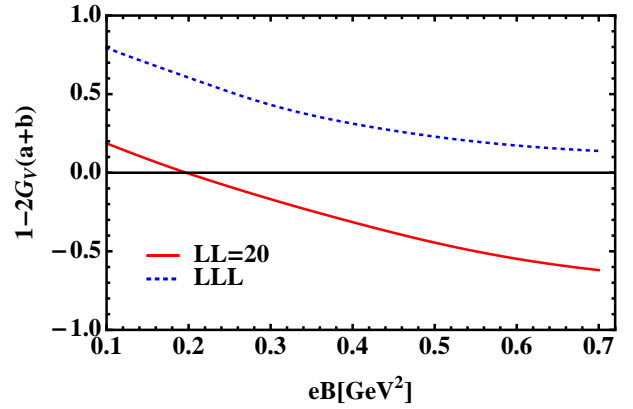


FIG. 8: The gap function  $1 - 2G_V(a + b)$  at  $M_{\rho^+} = 0$  as a function of  $eB$  with Landau levels  $LL=0$  and 20, respectively.

## V. CONCLUSION

After the prediction of the vacuum superconductor based on the the energy of a free particle under magnetic fields by neglecting the internal structure of vector mesons [8, 9], there are more efforts trying to investigate properties of charged  $\rho$  meson by considering the internal structure of vector mesons. However, most of these calculations tend to conclude that the mass of the charged vector meson  $\rho^\pm$  will not drop to zero at high magnetic field, therefore there would be no vacuum superconductor [22, 27, 28]. Therefore in this work, we carefully investigated the charged and neutral  $\rho$  meson mass in the presence of external magnetic field in the framework of NJL model.

In the NJL model, mesons are constructed by infinite sum of quark-loop chains by using random phase approximation. We calculate the  $\rho$  meson polarization tensor to the leading order of  $1/N_c$  expansion by considering one quark loop contribution, and solve the masses of vector meson with different spin component from gap equations. The constituent quark mass is also solved self-consistently under the magnetic field. It is found that the constituent quark mass increases with the magnetic field, which is famous magnetic catalysis effect. The masses of the neutral vector meson  $\rho^0$  with spin component  $s_z = 0, \pm 1$  and the charged vector meson  $\rho^\pm$  with  $s_z = 0$  also increases with magnetic field. However, the mass square of the charged vector meson  $\rho^+$  ( $\rho^-$ ) with  $s_z = +1$  ( $s_z = -1$ ) decreases linearly with magnetic field and drops to zero at the critical magnetic field  $eB_c \simeq 0.2\text{GeV}^2$ , which indicates the possible condensation of charged vector meson in the vacuum. This critical magnetic field is much lower than the value  $eB_c = 0.6\text{GeV}^2$  predicted by a point-like vector meson.

At the end, we analyze possible reasons why other groups [22, 27, 28] obtained different results on the charged  $\rho$  meson mass. One possible reason is that it might due to the lowest Landau level approximation as used in [27] (though we are not sure whether LLL approx-

imation is used in [22, 28]). We find that if we use lowest Landau level approximation, the mass of the charged vector meson  $\rho^\pm$  for  $s_z = \pm 1$  cannot drop to zero at high magnetic fields! Another reason might due to the spin decomposition of the  $\rho$  meson polarization tensor. In order to obtain conclusive results on the masses of charged vector meson under magnetic field, more efforts are needed in the future.

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### Appendix A: Integrals

For numerical calculations, we have defined the notations for the integrals of  $k_0$  as follows:

$$i \int \frac{dk_0}{2\pi} \frac{\bar{p}\bar{k} - M^2}{(p_0^2 - \omega_p^2)(k_0^2 - \omega_k^2)} = \frac{1}{2}(iI_1 + iI'_1) - \left[ \frac{1}{2} (M^2 - \bar{p}_2^2 - \bar{k}_2^2) + \bar{p}_2\bar{k}_2 \right] iI_2, \quad (\text{A1})$$

$$i \int \frac{dk_0}{2\pi} \frac{\bar{p}\bar{k} - M^2 + 2\bar{p}_2\bar{k}_2}{(p_0^2 - \omega_p^2)(k_0^2 - \omega_k^2)} = \frac{1}{2}(iI_1 + iI'_1) - \left[ \frac{1}{2} (M^2 - \bar{p}_2^2 - \bar{k}_2^2) - \bar{p}_2\bar{k}_2 \right] iI_2, \quad (\text{A2})$$

$$i \int \frac{dk_0}{2\pi} \frac{p_0k_0 + \bar{p}_2\bar{k}_2 + k_3^2 - M^2}{(p_0^2 - \omega_p^2)(k_0^2 - \omega_k^2)} = \frac{1}{2}(iI_1 + iI'_1) + \left[ 2k_3^2 - \frac{1}{2} (M^2 - \bar{p}_2^2 - \bar{k}_2^2) + \bar{p}_2\bar{k}_2 \right] iI_2, \quad (\text{A3})$$

$$i \int \frac{dk_0}{2\pi} \frac{p_0k_0 - \bar{p}_2\bar{k}_2 + k_3^2 - M^2}{(p_0^2 - \omega_p^2)(k_0^2 - \omega_k^2)} = \frac{1}{2}(iI_1 + iI'_1) + \left[ 2k_3^2 - \frac{1}{2} (M^2 - \bar{p}_2^2 - \bar{k}_2^2) - \bar{p}_2\bar{k}_2 \right] iI_2, \quad (\text{A4})$$

$$(\text{A5})$$

with

$$I_1 = \int \frac{dk_0}{2\pi} \frac{1}{k_0^2 - \omega_p^2}, \quad (\text{A6})$$

$$I'_1 = \int \frac{dk_0}{2\pi} \frac{1}{k_0^2 - \omega_k^2}, \quad (\text{A7})$$

$$I_2 = \int \frac{dk_0}{2\pi} \frac{1}{((k_0 + q_0)^2 - \omega_p^2)(k_0^2 - \omega_k^2)}. \quad (\text{A8})$$

Replacing the integral over  $k_0$  to Matsubara summa-

tion as in [42], one obtains:

$$iI_1 = \frac{\tanh[\frac{\omega_p}{2T}]}{2\omega_p}, \quad (\text{A9})$$

$$iI'_1 = \frac{\tanh[\frac{\omega_k}{2T}]}{2\omega_k}, \quad (\text{A10})$$

$$iI_2 = - \int dy \frac{\tanh[\frac{\sqrt{(-1+y)(M^2y - \omega_k^2) + y\omega_p^2}}{2T}]}{4[(-1+y)(M^2y - \omega_k^2) + M^2\omega_p^2]^{\frac{3}{2}}} \quad (\text{A11})$$

At zero temperature  $T = 0$ , we have

$$\tanh[\frac{\omega_p}{2T}] = 1, \quad (\text{A12})$$

$$\tanh[\frac{\omega_k}{2T}] = 1, \quad (\text{A13})$$

$$\tanh[\frac{\sqrt{(-1+y)(M^2y - \omega_k^2) + y\omega_p^2}}{2T}] = 1. \quad (\text{A14})$$

### Appendix B: The calculation of $\Pi_{ab}^{\mu\nu}$ in weak magnetic field

In this Appendix, we investigate whether the mass of charged  $\rho$  meson decreases with  $eB$  at weak magnetic field. In order to double check our results by using the Ritus propagator, here we use the quark propagator with the form in Ref. [16],

$$\tilde{S}(k) = i \exp(-\frac{\mathbf{k}_\perp^2}{|QeB|}) \sum_{n=0}^{\infty} (-1)^n \frac{D_n(QeB, k)}{k_0^2 - 2|QeB|n - k_3^2 - M^2} \quad (\text{B1})$$

with

$$D_n(QeB, k) = (k^0\gamma^0 - k^3\gamma^3 + m) [(1 - i\gamma^1\gamma^2 \text{sgn}(QeB))L_n(2\frac{\mathbf{k}_\perp^2}{|QeB|}) - (1 + i\gamma^1\gamma^2 \text{sgn}(QeB))L_{n-1}(2\frac{\mathbf{k}_\perp^2}{|QeB|})] + 4(k^1\gamma^1 + k^2\gamma^2)L_{n-1}(2\frac{\mathbf{k}_\perp^2}{|QeB|}). \quad (\text{B2})$$

where  $\mathbf{k}_\perp$  is  $(k^1, k^2)$ ,  $L_n$  is Laguerre polynomials and  $Q$  is a diagonal matrix with the entries  $q_f = \{2/3, -1/3\}$ .

The one-loop polarization of  $\rho$  meson is  $\Pi^{\mu\nu, ab} = -i \text{Tr}[\tilde{S}(k)\gamma^\mu\tau_a\tilde{S}(p)\gamma^\nu\tau_b]$ , where  $p = k + q$  and  $q$  is the momentum of  $\rho$ . We only calculate the polarization function of  $\rho^-$  in the rest frame,

$$\Pi^{11} = iN_c N_f \int \frac{dk^4}{(2\pi)^4} \exp\left(-\frac{9\mathbf{k}_\perp^2}{2|eB|}\right) \sum_{k=0}^{\infty} \sum_{p=0}^{\infty} (-1)^{p+k} \left[ 8 \left( L_k(3\frac{\mathbf{k}_\perp^2}{|eB|})L_p(6\frac{\mathbf{k}_\perp^2}{|eB|}) + L_{k-1}(3\frac{\mathbf{k}_\perp^2}{|eB|})L_{p-1}(6\frac{\mathbf{k}_\perp^2}{|eB|}) \right) \right] \frac{k_0p_0 - k_3^2 - M^2}{(k_0^2 - \frac{4}{3}|eB|k - k_3^2 - M^2)(p_0^2 - \frac{2}{3}|eB|p - p_3^2 - M^2)}, \quad (\text{B3})$$



$$\Pi^{12} = iN_c N_f \int \frac{dk^4}{(2\pi)^4} \exp\left(-\frac{9\mathbf{k}_\perp^2}{2|eB|}\right) \sum_{k=0}^{\infty} \sum_{p=0}^{\infty} (-1)^{p+k} \left[ 8i \left( L_{k-1} \left( 3 \frac{\mathbf{k}_\perp^2}{|eB|} \right) L_{p-1} \left( 6 \frac{\mathbf{k}_\perp^2}{|eB|} \right) - L_k \left( 3 \frac{\mathbf{k}_\perp^2}{|eB|} \right) L_p \left( 6 \frac{\mathbf{k}_\perp^2}{|eB|} \right) \right) \right] \frac{k_0 p_0 - k_3^2 - M^2}{(k_0^2 - \frac{4}{3}|eB|k - k_3^2 - M^2)(p_0^2 - \frac{2}{3}|eB|p - p_3^2 - M^2)}. \quad (\text{B4})$$

Combing Feynman parameter for the denominator factor and the proper time representation [45], we obtain [46]

$$\frac{1}{ab} = \int_0^1 dx \int_0^\infty d\tau \tau \exp[(xa + (1-x)b)\tau]. \quad (\text{B5})$$

With the help of generating function of Laguerre polynomials [47]:

$$\sum_{t=0}^{\infty} t^n L_{n-i}^a(\xi) = \frac{t^i}{(1-t)^{a+1}} \exp\left[\frac{-t\xi}{1-t}\right], \quad (\text{B6})$$

for  $|t| < 1$ , we can calculate the summation of Landau level directly.

Performing Taylor expansion at  $eB = 0$ , we set  $M_{\rho^-}^2(eB) = M_{\rho^-}^2(eB=0) - \kappa eB$ , and we obtain  $\kappa \approx 2$  for spin component  $s_z = 1$  numerically by using the relation in (40). The same result can be obtained for  $\rho^+$  meson.

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